Pendulum Snake “Snack”
Math Root- Worksheet

Part 1
What to do?
Calculate the lengths of each pendulum using your math skills.

The longest pendulum on this “snack” is 99.4 cm (Measured to the center of the mass of the hex nut to the string underneath the peg board.) The pendulum will swing back and forth 15 times in 30 seconds.

(L)-Length = 99.4 cm
(N)- Number of back and forth wings in 30 seconds

N=15
L=99.4 cm

Subsequent lengths can be calculated by using the following formula:

$L_{n+1} = L_n (N/N+1)^2$

For example:

$L_{16} = L_{15} (15/16)^2$

Length of the pendulum for 16 would be:
N=16
L_{16}=99.4cm $(15/16)^2$
=99.4$(0.9375)^2$
=99.4(.8789)
=87.3 cm

Table 1

Length (L) cm	Number of swings in 30 sec (N)
99.4	15
87.3	16
	17
	18
	19
	20
	21
	22
	23
	24

Answers -Table 1

Length (L) cm	Number of swings in 30 sec (N)
99.4	15
87.3	16
77.4	17
69.0	18
61.9	19
55.9	20
50.7	21
46.2	22
42.3	23
38.8	24
35.8	25
33.1	26
31.0	27
28.1	28

Part 2:

What to do?
Notice the number of pendulums (10) and the length of each pendulum is different. Using a long rod or meter stick rest all the pendulums on the stick to stop any motion. Try to release them all at the same time.
Watch!
What do you notice about the pendulums motion over time? 30 seconds?

The pendulums move together at the start and then some begin to change position relative to their neighbor, creating the snake pattern. Eventually, all apparent patterns are lost until suddenly, every other pendulum is moving opposite its neighbors wait longer and all of the pendulums return and move together in one line.

The pendulums go from 15 swings in 30 seconds to 24 swings in 30 seconds.

Even and Odd:
What happens after 15 seconds? Time the pendulums and at 15 seconds notice the position of the pendulums. What pendulums are opposite?

After 15 seconds all of the even number pendulums will have finished an integer number of swings while all odd ones will have completed a half-a-swing. This creates the pattern in which each pendulum is moving opposite to its neighbor.

Differences:
Look at just two pendulums, 15 and 16. Notice that when they start together, after 30 seconds they return to swing together only once at the end of the interval. The difference between 15 and 16 is 1.

Now, look at pendulums 16 and 18, also 15 and 17. Both pairs swing together twice in 30 seconds. The even number pair will match up half way through the 30 seconds. Half way through they have made an integer number of swings, half of 16 is 8 and half of 18 is 9. The odd number pendulums also match half way through, except each match on a half swing. Half of 15 is 7.5 and half of 17 is 8.5.
Each of these pairs will swing together 2 times during the 30 seconds.

Pendulums 15 and 18 will swing together 3 times in 30 seconds, after 10 seconds, 20 seconds and 30 seconds. In 10 seconds the 15 will swing 5 times and the 18 will swing 6 times. The greatest common factor (GCF) is 3, so they will swing together 3 times in 30 seconds and will have an integer number of swings.

Pendulums 16 and 19 also swing together 3 times in 30 seconds, but not after integer number of swings, after 10 seconds 16 will swing 5 1/3 times and 19 will swing 6 1/3 times.
They do not have a GCF.

The number difference in the number of swings in 30 seconds between pendulums give the number of times they repeat their pattern in 30 seconds. Only pendulums with the GCF will repeat after an integer number of swings.

16 and 20 have a GCF of 4 so they will swing together 4 times in 30 seconds.

What about 18 and 24? 15 and 20? 6 and 5

Part 3:

Frequency of a pendulum, F, is oscillations per second and is inversely proportional to the square root of the length of the pendulum, L.

15 swings in 30 seconds or 15/30 is the frequency, to find the period of 15/30, use the reciprocal of 30/15, the period is 2 seconds. So T= 2 seconds. Which is almost a meter in length.

16 swings in 30 seconds or 16/30 is the frequency, the reciprocal 30/16 and the period is 1.875 seconds. So T = 1.875 seconds.
Complete Table 2 with data from Table 1.

Table 2

	Frequency in 30 sec	Length (L) cm Measured	Length (L) cm Calculated	Period T sec
	15	99.4		2
	16	87.3		1.875

Math Root 2

L= length
g = acceleration due to gravity 9.8 m/s/s or 9.8 m/s^2
T = period of the pendulum
f = oscillations per second

Frequency of the pendulum $f = ½\, \pi\sqrt{(g/L)}$
Period of a pendulum $T= 2\pi\sqrt{(L/g)}$

Length of pendulum # 15 is 99.4 cm
Convert 99.4 cm to .994 meters

$T= 2(3.14)\sqrt{(.994\ m/9.8\ m/s^2)}$ (meters factor out)
T= (6.28) (3.18)s
T=2 s

Let's do one more:

Length of pendulum # 21 is 50.7 cm
Convert to meters = .507 m
$T= 2(3.14)\sqrt{(.507\ m/9.8m/s^2)}$ (meters factor out)
T= (6.28) (.227)s
T= 1.42 s
T= 30/21 = 1.42 s

Comparing data on Table 2
Make a graph:

Compare the periods for the shorter pendulum with the period of the longer pendulum. How does the period change when you double the length of the pendulum? The period of the shorter pendulum is 1.4 seconds, while the period of the longer pendulum is 2 seconds. When you double the length, the period increases times 1.4 and 1.4 is just about the square of 2.

Graph:
Period versus Length using the table

To see if it really is a square root function, plot the period versus the square root of the length. If the period is proportional to the square root of the length, the data will lie on a straight line.

Summary and conclusion:
The frequency of a pendulum, F, in oscillations per second is inversely proportional to the square root of the length of the pendulum, L.

$f\ \alpha\ 1/(L)\ 0.5$

Thus if we number the pendulums by their frequency, e.g. f =1 per 30 seconds, f=n per 30 seconds,
Then the length of the pendulum number n is $L\ \alpha\ 1/f^2\ \alpha\ 1/n^2$.

The series of lengths of pendulum number n goes inversely proportional to n squared.

The equation is $f = 1/(2\pi)\sqrt{(g/L)}$ or $T = 2\pi\sqrt{(L/g)}$, because the frequency of a pendulum represents the number of vibrations per second. This quantity is measure in hertz (hz) and is the reciprocal of the pendulum's period.
(This equation works well with small 10-15 °angle swings.)

Adapted by Kathy Holt from the Scientific Explorations with Paul Doherty.
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